

Alpha-decay energies of superheavy nuclei for the Fayans functional

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Alpha-decay energies for several chains of super-heavy nuclei are calculated by using Fayans functional FaNDF⁰. They are compared to the experimental data and predictions of two Skyrme functionals, SLy4 and SkM*, and of the macro-micro method as well. The corresponding lifetimes are calculated with the use of the semi-phenomenological formulas by Parkhomenko and Sobiczewski and by Royer and Zhang.

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I. INTRODUCTION

During last decade, remarkable progress has been achieved in a synthesis of superheavy nuclei belonging to the so-called “stability island”, the elements with charge numbers $Z \simeq 114 - 120$, predicted about fifty years ago [1, 2]. Two isotopes of the element with $Z = 116$, named later Lv, and one with $Z = 118$ were created first by Oganessian *et. al.* [3]. After the element 118 two isotopes of the element $Z = 117$ were synthesized [4]. The latter element was investigated in more details in Refs. [5–8]. An attempt [9] to create the element with $Z = 120$ turned out to be unsuccessful, however, evidently, efforts in this direction will be continued.

In fact, the stability island turned out to be “the stability shallow”, as all of these new nuclei are not stable. Although their lifetimes are large in the “nuclear scale”, they are usually not sufficiently long lived to be detected in a usual way. All of them undergo alpha-decay, and the authors of the experimental works cited above detected, for each primary nucleus, a chain of several alpha-decays by measuring the alpha-decay energies E_α with high accuracy, and the respective lifetimes T_α , leading to a nucleus which was already known. Therefore, a theoretical support is desirable to make such kind of indirect identification of the new superheavy nuclei more reliable.

For even-even nuclei, in which the transition occurs between the 0^+ ground states, the alpha-decay energies $E_\alpha(Z, N)$ are determined, with allowance for the recoil

effect, in terms of the mass difference between nuclei related by alpha decay:

$$Q_\alpha(Z, N) = M({}_Z^AX_N) - M({}_{Z-2}^{A-4}Y_{N-2}) - M_\alpha. \quad (1)$$

For odd and odd-odd nuclei, a correction to this simple formula could appear due to a possible excitation of the parent and/or daughter nucleus which may occur in the real experimental situation.

In Refs. [3–7], the experimental data for Q_α were compared with predictions [10, 11] of the so-called macro-micro method (MMM) [12, 13]. In this method, the binding energy of a nucleus is found as the sum of two terms, a macroscopic energy which was calculated on the basis of the liquid-drop model and a shell correction energy which was found according to the Strutinsky method [2]. In general, such a comparison confirmed the identification of new superheavy nuclei. In Ref. [14], a partially self-consistent calculation of Q_α for new superheavy α -chains was carried out on the base of the finite-range energy density functional (EDF) by Fayans *et al.* [15–17], but the deformation energy was considered in an approximate, in fact, non-consistent way. The total binding energy of a deformed nucleus was presented as a sum of two terms, $E_B(N, Z) = E_{\text{sph}}(N, Z) + E_{\text{def}}(N, Z)$, where the main, spherical one, was found for a Fayans EDF, whereas the “deformation” addendum was taken from published tables for two Skyrme EDFs. To be definite, a version DF3-a [18] of the initial Fayans EDF DF3 [16, 17] was used for the spherical energy. For the deformation energy, the EDF HFB-17 [19, 20] was used for nuclei with

$Z \leq 110$ and MSk7 [21], for $Z > 110$. Accuracy of such semi-self-consistent calculations turned out to be only a bit worse compared to that of the MMM approach.

Absence of a deformed Fayans EDF solver was the reason of the use of such non-consistent ansatz in Ref. [14]. Recently, however, such a code has been developed [22]. A localized version FaNDF⁰ [23] of the general finite range Fayans EDF was used, making the surface term more similar to the Skyrme one. This allowed to employ, with some modifications, the computer code HF-BTHO [24], developed originally for Skyrme EDFs. First applications of this code to deformed nuclei [22, 25–27] with the use of the original set of parameters of the EDF FaNDF⁰ [23] found for spherical nuclei turned out to be rather successful. The goal of the present work is to provide a new, completely self-consistent, calculation of Q_α energies for six superheavy α -chains with the Fayans EDF. For a comparison, calculations with two popular Skyrme EDFs, SLy4 and SkM*, are carried out. Predictions of the self-consistent methods are compared to the MMM of Refs. [10, 11].

Theoretical predictions for Q_α are important not only by itself but also to find the lifetime T_α . Indeed, the latter is governed mainly by the exponential Gamow factor [28], which is found almost unambiguously in terms of Q_α by finding the penetrability of the Coulomb barrier in the daughter nucleus for an emitted alpha particle. Unfortunately, at the present there is no a reliable microscopic theory for the pre-exponential factor as this is a very complicated many-body problem. It is worth to mention recent studies in this direction [29–32]. Some of them are rather promising but do not provide a simple tool for systematic calculations. In such a situation, the phenomenological approaches are more practical. Most of them are close to the classical Viola-Seaborg formula [33], which involves seven phenomenological parameters. Here, just as in [14], we use the five-parameter modification of this formula [34] by Parkhomenko and Sobiczewski (PS). For a comparison, similar calculations were repeated with the use of more recent 12-parameter formula by Royer and Zhang (RZ) [35].

II. FAYANS EDF PREDICTIONS FOR Q_α VALUES IN SUPERHEAVY NUCLEI

In this section, we give predictions of the Fayans EDF FaNDF⁰ [23] for six α -decay chains which begin from the following parent superheavy nuclei: $^{294}118$, $^{294}117$, $^{293}117$, $^{291}117$, $^{288}115$, and $^{287}115$. For completeness, we write down explicitly main ingredients of this EDF. In the Fayans method, the ground state energy of a nucleus is considered as a functional of normal density ρ and anomalous density ν , as

$$E_0 = \int \mathcal{E}[\rho(\mathbf{r}), \nu(\mathbf{r})] d^3r, \quad (2)$$

where the isotopic indices and the spin-orbit densities are omitted for brevity.

The volume part of the EDF, $\mathcal{E}^v(\rho)$, is taken as a fractional function of densities $\rho_+ = \rho_n + \rho_p$ and $\rho_- = \rho_n - \rho_p$:

$$\mathcal{E}^v(\rho) = C_0 \left[a_+^v \frac{\rho_+^2}{4} f_+^v(x) + a_-^v \frac{\rho_-^2}{4} f_-^v(x) \right], \quad (3)$$

where

$$f_+^v(x) = \frac{1 - h_{1+}^v x^\sigma}{1 + h_{2+}^v x^\sigma} \quad (4)$$

and

$$f_-^v(x) = \frac{1 - h_{1-}^v x}{1 + h_{2-}^v x}. \quad (5)$$

Here, $x = \rho_+/\rho_0$ is the dimensionless nuclear density, $\rho_0 = 2(k_F^0)^3/3\pi^2$ being the equilibrium symmetric nuclear matter density. The coefficient $C_0 = (dn/d\varepsilon_F)^{-1} = \pi^2/(k_F^0 m)$ is the usual normalization factor used in the theory of finite Fermi system (TFFS) [36], the inverse density of states at the Fermi surface. The power parameter $\sigma = 1/3$ is chosen in the FaNDF⁰ functional, in contrast to the case for DF3 or DF3-a, where $\sigma = 1$ is used. The dimensionless parameters in Eqs. 3–5 are the same as in [23]: $a_+^v = -9.559$, $h_{1+}^v = 0.633$, $h_{2+}^v = 0.131$, $a_-^v = 4.428$, $h_{1-}^v = 0.25$, and $h_{2-}^v = 1.300$. They correspond to the following characteristics of nuclear matter: the equilibrium density $\rho_0 = 0.160 \text{ fm}^{-3}$ (the corresponding mean radius parameter is $r_0 = 1.143 \text{ fm}$), the energy per particle $\mu = -16.0 \text{ MeV}$, the incompressibility $K_0 = 220 \text{ MeV}$, and the asymmetry energy coefficient of $a_{\text{sym}} = 30.0 \text{ MeV}$. Higher derivatives of the EDF of nuclear matter over the densities ρ_+ , ρ_- , suggested in [37] for the Skyrme EDFs as a set of criteria for functionals, are given in [22] for the FaNDF⁰ functional.

If one sets $h_{2+}^v = h_{2-}^v = 0$, the volume part of the Fayans EDF reduces to the form typical for Skyrme EDFs. As it was discussed in detail in [22], the “Fayans denominator” and the use of the bare mass $m^* = m$, both peculiarities of the Fayans EDF method, are generically related to the self-consistent TFFS [38], reflecting in a hidden form the energy-dependence effects inherent to this approach.

Until recently, the Fayans EDF was applied to spherical nuclei only. These applications were rather successful, in comparison with analogous Skyrme HFB calculations. They included the analysis of the magnetic [39, 40] and quadrupole [41, 42] moments in odd nuclei, of characteristics of the first 2^+ excitations in even semi-magic nuclei [43, 44], of charge radii [45], and of beta-decay [46] as well. In addition, single-particle spectra of seven magic nuclei was described with high accuracy [47]. A short review comparison of predictions of Fayans and Skyrme EDFs for these phenomena in spherical nuclei was given in Ref. [48].

The use of the local approximation for the Yukawa finite-range function, $\text{Yu}(r) \rightarrow 1 - r_c^2 \nabla^2$, in the DF3-like

EDFs leads to the following structure of the surface term of the FaNDF⁰ functional:

$$\mathcal{E}^s(\rho) = C_0 \frac{1}{4} \frac{a_+^s r_0^2 (\nabla \rho_+)^2}{1 + h_+^s x^\sigma + h_\nabla^s r_0^2 (\nabla x_+)^2}, \quad (6)$$

with $h_+^s = h_{2+}^v$, $a_+^s = 0.600$, $h_\nabla^s = 0.440$. This approximation is the main point which permitted in [22] to modify the Skyrme HFB computer code HFBTHO [24] for the Fayans EDF.

Here, just as in [22], we use a two-parameter form for the anomalous term of the EDF:

$$\mathcal{E}_{\text{anom}} = C_0 \sum_{i=n,p} \nu_i^\dagger(\mathbf{r}) f_i^\xi(x_+(\mathbf{r})) \nu_i(\mathbf{r}), \quad (7)$$

where the density-dependent dimensionless effective pairing force is

$$f^\xi(x_+) = f_{\text{ex}}^\xi + h^\xi x_+. \quad (8)$$

Two models for pairing will be used, the volume one with $h^\xi = 0$, and the surface model, with $h^\xi \simeq -f_{\text{ex}}^\xi$. As we use the zero-range pairing force, a cut-off should be used in solving the gap equation. Here we use the same cut-off energy $E_{\text{cut}} = 60$ MeV as in [22]. Lastly, for the odd nuclei, we used equal filling approximation.

Pairing interaction influences significantly on the one-neutron separation energies

$$S_n(N, Z) = B(N, Z) - B(N-1, Z). \quad (9)$$

In figure 1, they are shown for uranium isotopes, calculated with the FaNDF⁰ EDF, with two models of pairing specified above. The corresponding values of parameters of Eq. (8) are ($f^\xi = -0.440$, $h^\xi = 0$) for the volume pairing and ($f^\xi = -1.433$, $h^\xi = 1.375$) for the surface one. In this work, the calculation scheme for the Fayans EDF is the same as that in Ref. [22] for the uranium chain. In particular, the HFB equations were solved in a basis of 25 oscillator shells. Comparison is made with experimental data [49] and predictions from the Skyrme EDF HFB-17 EDF [19, 20]. We see that the Fayans EDF with both pairing models reproduce experimental data sufficiently well, with accuracy comparable with that of the Skyrme EDF HFB-17.

Next, we investigate the alpha-decay energies Q_α . The results of calculations are presented in figure 2 and in Table I. For the Fayans EDF, predictions of the volume and surface pairing are given. A comparison is made with the data and predictions of two Skyrme EDFs, the SLy4 [50] and SkM* [51]. Corresponding results are found by us with the use of the code [24]. In this case, 20 oscillator shells were used, in accordance with [52]. For both of the Skyrme EDFs, the mixed pairing is used [24]:

$$V_{\text{pair}}^{n,p}(\mathbf{r}, \mathbf{r}') = V_0^{n,p} \left(1 - \alpha \frac{\rho(\mathbf{r})}{\rho_c} \right) \delta(\mathbf{r} - \mathbf{r}'), \quad (10)$$

$\alpha = 0.5$, with obvious notation. The pairing strength is taken to be the same for neutrons and protons, namely

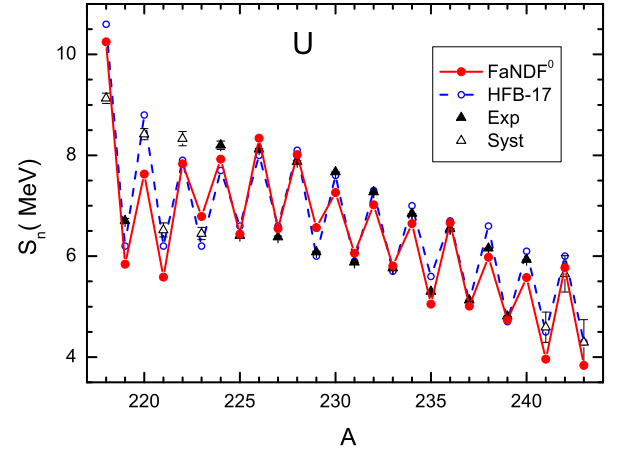


FIG. 1: The S_n values for the U isotopic chain.

$V_0^n = V_0^p = -259.0$ MeV. For completeness, we included also predictions of the MMM method taken from tables in [10], $Z = 102-109$, and [11], $Z = 110-120$. Several empty places in the MMM column of Table I are caused with absence of the corresponding values in the tables cited above.

A remark should be made about the experimental Q_α values used in the tables and figures 2–7. They are taken mainly from the references cited in the caption of Table I, where the alpha-energies E_α were measured, and Q_α being found by accounting the recoil effect. This recipe may be doubtful in the case of odd and odd-odd nuclei where the α -transition between ground states can be forbidden, and the observed transition may connect excited states. The analysis of the lifetimes T_α may help to clear up this point. They will be analyzed in the next Section. There are also cases when experimental Q_α values are not available. In this kind of situation we use estimated masses from mass systematics of Ref. [49]. They are shown with the open triangles in the figures.

By eye, the accuracy of the MMM is higher compared to self-consistent approaches. Among the latest EDFs, the Fayans one, with both the models for pairing, has the accuracy comparable with both the Skyrme ones. To estimate it quantitatively, we present in Table II the differences

$$\delta Q_\alpha = Q_\alpha^{\text{theor}} - Q_\alpha^{\text{exp}}, \quad (11)$$

with obvious notation. In the last column, the mark “(syst)” is used for the mass systematics data given in Ref. [49]. In the last line of the table, values are given of the root-mean-square deviation (RMSD) of the theory under consideration from the experiment:

$$\langle \delta Q_\alpha \rangle_{\text{rms}} = \sqrt{\frac{1}{N} \sum_i (Q_{\alpha,i}^{\text{theor}} - Q_{\alpha,i}^{\text{exp}})^2}. \quad (12)$$

From the δQ_α results, we can see that the MMM exceeds in accuracy all the self-consistent methods used,

TABLE I: α -decay energies Q_α (MeV) of superheavy nuclei. In each line, Z and A correspond to the parent nucleus. FaNDF⁰(surf) and FaNDF⁰(vol) denote the Fayans EDF with the surface and volume pairing, correspondingly. Experimental data are taken from [3] ^(a), [8] ^(b), [7] ^(c), [49], compilation of several experiments, ^(d), [53] ^(e). The estimated data (^(syst)) are taken from the systematics in [49].

Nuclei	FaNDF ⁰ (surf)	FaNDF ⁰ (vol)	SLy4	SkM*	MMM	Exp.
²⁹⁴ 118	11.278	11.310	11.364	11.330	12.11	11.81 (0.06) ^{a)}
²⁹⁰ Lv	11.116	10.981	10.354	11.074	11.08	10.99 (0.08) ^{a)}
²⁸⁶ Fl	10.847	10.683	9.719	11.734	10.86	10.37 (0.06) ^{a)}
²⁸² Cn	11.188	11.005	10.886	11.688	10.68	10.11 (0.2) ^(syst)
²⁷⁸ Ds	11.272	11.026	10.849	11.802	10.54	10.37 (0.2) ^(syst)
²⁷⁴ Hs	9.497	9.788	9.870	9.932	9.55	9.57 (0.2) ^(syst)
²⁷⁰ Sg	8.913	9.407	9.004	9.319	8.74	8.99 (0.3) ^(syst)
²⁶⁶ Rf	7.617	7.316	7.052	8.508	7.05	7.55 (0.3) ^(syst)
²⁶² No	7.497	7.210	6.649	7.932	6.86	7.25 (0.3) ^(syst)
²⁹⁴ 117	10.225	10.131	11.629	11.033	11.43	11.20 (0.04) ^{b)}
²⁹⁰ 115	11.117	10.898	10.143	10.792	10.65	10.45 (0.04) ^{b)}
²⁸⁶ 113	9.494	9.479	8.792	10.879	9.98	9.79 (0.05) ^{c)}
²⁸² Rg	10.524	10.701	10.037	11.512	9.85	9.18 (0.03) ^{b)}
²⁷⁸ Mt	10.334	9.681	9.851	10.442	9.55	9.59 (0.03) ^{b)}
²⁷⁴ Bh	8.957	9.187	8.911	9.285	8.83	8.97 (0.03) ^{b)}
²⁷⁰ Db	7.966	7.528	7.643	8.376	8.34	8.02 (0.03) ^{b)}
²⁶⁶ Lr	7.717	8.663	7.993	8.449	6.79	7.57 (0.3) ^(syst)
²⁶² Md	6.564	6.431	6.011	7.621	-	6.5 (0.3) ^(syst)
²⁹³ 117	10.605	10.502	10.976	10.990	11.53	11.18 (0.05) ^{d)}
²⁸⁹ 115	11.356	11.167	10.035	10.688	10.74	10.52 (0.05) ^{d)}
²⁸⁵ 113	10.271	10.154	9.592	11.900	10.21	10.03 (0.05) ^{d)}
²⁸¹ Rg	10.850	11.003	10.719	11.736	10.48	9.41 (0.05) ^{c)}
²⁷⁷ Mt	10.498	9.956	10.007	10.618	9.84	9.71 (0.2) ^(syst)
²⁷³ Bh	9.014	9.456	9.170	9.386	8.89	9.06 (0.3) ^(syst)
²⁶⁹ Db	8.596	8.739	7.902	8.305	8.17	8.49 (0.3) ^(syst)
²⁶⁵ Lr	7.162	7.021	7.208	8.503	6.62	7.23 (0.2) ^(syst)
²⁶¹ Md	6.708	6.960	6.400	8.030	-	6.75 (0.4) ^(syst)
²⁹¹ Lv	10.959	10.987	10.557	10.798	10.91	10.89 (0.07) ^{a)}
²⁸⁷ Fl	10.463	10.243	9.130	11.528	10.56	10.16 (0.06) ^{a)}
²⁸³ Cn	10.506	10.275	10.499	11.731	10.16	9.67 (0.06) ^{a)}
²⁷⁹ Ds	11.097	10.876	10.234	11.361	10.24	9.84 (0.06) ^{a)}
²⁷⁵ Hs	9.321	9.606	9.600	9.920	9.41	9.44 (0.06) ^{a)}
²⁷¹ Sg	8.747	8.115	8.554	8.541	8.71	8.67 (0.08) ^{a)}
²⁶⁷ Rf	7.998	9.077	8.046	8.204	-	7.89 (0.3) ^(syst)
²⁶³ No	7.031	6.759	6.287	8.634	6.45	7.0 (0.4) ^(syst)
²⁸⁸ 115	11.376	10.968	9.389	11.485	10.95	10.61 (0.06) ^{e)}
²⁸⁴ 113	10.661	10.588	10.393	11.930	10.68	10.15 (0.06) ^{e)}
²⁸⁰ Rg	11.208	11.274	11.023	11.717	10.77	9.87 (0.06) ^{e)}
²⁷⁶ Mt	10.681	10.393	10.490	10.919	10.09	9.85 (0.06) ^{e)}
²⁷² Bh	8.978	9.612	8.836	9.128	9.08	9.02 (0.06) ^{e)}
²⁶⁸ Db	8.598	8.245	8.514	8.239	7.90	8.26 (0.3) ^{e)}
²⁶⁴ Lr	7.343	7.334	6.606	8.902	6.84	7.4 (0.3) ^(syst)
²⁶⁰ Md	6.823	7.259	6.919	8.189	-	6.94 (0.3) ^(syst)
²⁸⁷ 115	11.628	11.362	9.840	11.772	11.21	10.59 (0.09) ^{e)}
²⁸³ 113	11.264	11.090	10.990	12.269	11.12	10.12 (0.09) ^{e)}
²⁷⁹ Rg	10.494	10.607	11.489	12.107	11.08	10.37 (0.16) ^{e)}
²⁷⁵ Mt	11.789	11.726	10.585	10.909	10.34	10.33 (0.09) ^{e)}
²⁷¹ Hs	9.010	9.760	9.557	9.425	9.07	9.49 (0.16) ^(syst)
²⁶⁷ Db	8.312	7.667	7.412	8.378	7.41	7.9 (0.3) ^(syst)
²⁶³ Lr	7.742	7.355	6.979	8.744	7.26	7.68 (0.2) ^(syst)
²⁵⁹ Md	6.968	7.737	7.091	8.352	-	7.11 (0.2) ^(syst)

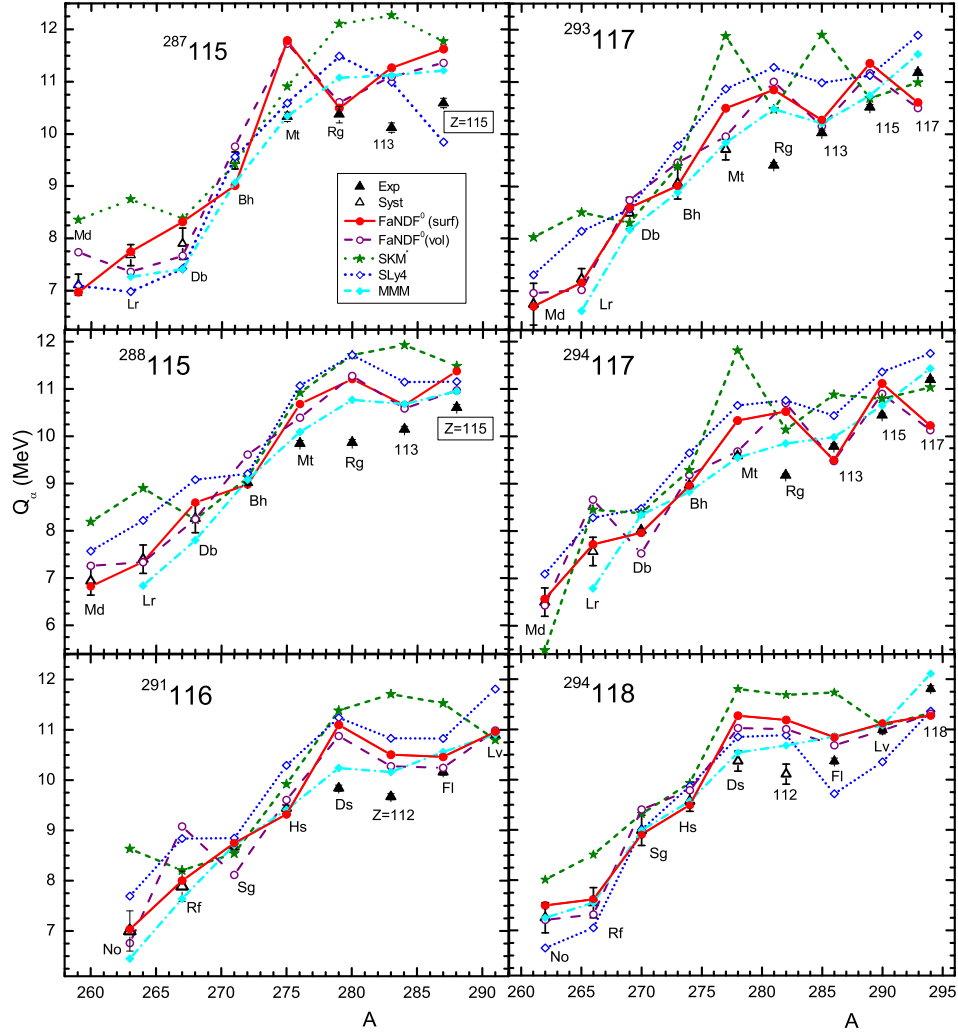


FIG. 2: The Q_α values for alpha decay chains starting from nuclei $^{287}_{115}$, $^{293}_{117}$, $^{288}_{115}$, $^{294}_{117}$, $^{291}_{116}$, and $^{294}_{118}$. Theoretical results are shown for FaNDF⁰ with surface and volume pairing, for SkM* and SLy4 Skyrme EDFs, and for MMM model. These are compared to experimental and estimated values.

with its RMSD being smaller by a factor of 1.5–2. However, it is worth to note that the MMM parameters, those of the liquid-drop model and of the Saxon-Woods shell-model potential, were fitted in Ref. [10, 11] to characteristics of heavy deformed nuclei in the uranium region, close to the nuclear map region under consideration. Per contra, the main part of the EDFs parameters is universal for all nuclei. Among the self-consistent calculations presented, the SLy4 EDF results in the highest accuracy. The Fayans EDF has approximately 10 % worse RMSD. However, it is worth to remind that the FaNDF⁰ parameters were fitted in [23] to characteristics of the spherical nuclei only of the region between calcium and lead chains. Evidently, more fine tuning of this EDF parameters is necessary, including heavy deformed nuclei into the fitting procedure. Accuracy of the SkM* EDF is approximately two times worse comparing to other EDFs used.

III. LIFETIMES T_α WITH RESPECT TO ALPHA-DECAY

For completeness we recite the commonly used classical formula by Viola and Seaborg [33]. In particular, it was used in [3]–[6] to connect values of Q_α and T_α found experimentally. It reads:

$$\lg T_\alpha(Z, N) = (aZ + b)Q_\alpha^{-1/2} + (cZ + d) + h_i, \quad (13)$$

where a, b, c, d, h_i are adjusted parameters. Three of them, $h_i, i = p, n, pn$, are introduced to reproduce a change of the α -decay lifetime in odd-proton, odd-neutron and odd-odd nuclei with respect to “favored” decays of even-even nuclei with zero orbital moment l of the α -particle. The ground states of mother and daughter odd or odd-nuclei have usually different J^π values, therefore the α -transition between them will be unfavored type with $l > 0$, with additional hindrance due

TABLE II: Differences in Eq. (11) between theoretical predictions of α -decay energies of superheavy nuclei from Table I and the corresponding experimental data. The mark “(syst)” in the last column denotes the data from the systematics in [49].

Nuclei	FaNDF ⁰ (surf)	FaNDF ⁰ (vol)	SLy4	SkM*	MMM	data from
²⁹⁴ 118	-0.532	-0.500	-0.446	-0.480	0.300	[3]
²⁹⁰ Lv	0.126	-0.009	-0.636	0.084	0.090	[3]
²⁸⁶ Fl	0.477	0.313	-0.651	1.364	0.490	[3]
²⁸² Cn	1.078	0.895	0.776	1.578	0.570	[49](syst)
²⁷⁸ Ds	0.902	0.655	0.479	1.432	0.170	[49](syst)
²⁷⁴ Hs	-0.073	0.218	0.300	0.362	-0.020	[49](syst)
²⁷⁰ Sg	-0.078	0.417	0.014	0.329	-0.250	[49](syst)
²⁶⁶ Rf	0.067	-0.234	-0.498	0.958	-0.500	[49](syst)
²⁶² No	0.247	-0.040	-0.601	0.682	-0.390	[49](syst)
²⁹⁴ 117	-0.975	-1.069	0.429	-0.167	0.230	[8]
²⁹⁰ 115	0.667	0.448	-0.307	0.342	0.200	[8]
²⁸⁶ 113	-0.296	-0.311	-0.998	1.089	0.190	[7]
²⁸² Rg	1.344	1.521	0.857	2.332	0.670	[8]
²⁷⁸ Mt	0.744	0.091	0.261	0.852	-0.040	[8]
²⁷⁴ Bh	-0.013	0.217	-0.059	0.315	-0.140	[8]
²⁷⁰ Db	-0.054	-0.492	-0.377	0.356	0.320	[8]
²⁶⁶ Lr	0.147	1.093	0.423	0.879	-0.780	[49](syst)
²⁶² Md	0.064	-0.069	-0.489	1.121	-	[49](syst)
²⁹³ 117	-0.575	-0.678	-0.204	-0.190	0.350	[49]
²⁸⁹ 115	0.836	0.647	-0.485	0.168	0.220	[49]
²⁸⁵ 113	0.241	0.124	-0.438	1.870	0.180	[49]
²⁸¹ Rg	1.440	1.593	1.309	2.326	1.070	[7]
²⁷⁷ Mt	0.788	0.246	0.297	0.908	0.130	[49](syst)
²⁷³ Bh	-0.046	0.396	0.110	0.326	-0.170	[49](syst)
²⁶⁹ Db	0.106	0.249	-0.588	-0.185	-0.320	[49](syst)
²⁶⁵ Lr	-0.068	-0.209	-0.022	1.273	-0.610	[49](syst)
²⁶¹ Md	-0.042	0.210	-0.350	1.280	-	[49](syst)
²⁹¹ Lv	0.069	0.097	-0.333	-0.092	0.020	[3]
²⁸⁷ Fl	0.303	0.083	-1.030	1.368	0.400	[3]
²⁸³ Cn	0.836	0.605	0.829	2.061	0.490	[3]
²⁷⁹ Ds	1.257	1.036	0.394	1.521	0.400	[3]
²⁷⁵ Hs	-0.119	0.166	0.160	0.480	-0.030	[3]
²⁷¹ Sg	0.077	-0.555	-0.116	-0.129	0.040	[3]
²⁶⁷ Rf	0.108	1.187	0.156	0.314	-	[49](syst)
²⁶³ No	0.031	-0.241	-0.713	1.634	-0.550	[49](syst)
²⁸⁸ 115	0.766	0.358	-1.221	0.875	0.130	[53]
²⁸⁴ 113	0.511	0.438	0.243	1.780	0.530	[53]
²⁸⁰ Rg	1.338	1.404	1.153	1.847	0.900	[53]
²⁷⁶ Mt	0.831	0.543	0.640	1.069	0.240	[53]
²⁷² Bh	-0.042	0.592	-0.184	0.108	0.060	[53]
²⁶⁸ Db	0.338	-0.015	0.254	-0.021	-0.360	[53]
²⁶⁴ Lr	-0.057	-0.066	-0.794	1.502	-0.560	[49](syst)
²⁶⁰ Md	-0.117	0.319	-0.021	1.249	-	[49](syst)
²⁸⁷ 115	1.038	0.772	-0.750	1.182	0.620	[53]
²⁸³ 113	1.144	0.970	0.870	2.149	1.000	[53]
²⁷⁹ Rg	0.124	0.237	1.119	1.737	0.710	[53]
²⁷⁵ Mt	1.459	1.396	0.255	0.579	0.010	[53]
²⁷¹ Bh	-0.480	0.270	0.067	-0.065	-0.420	[49](syst)
²⁶⁷ Db	0.412	-0.233	-0.488	0.478	-0.490	[49](syst)
²⁶³ Lr	0.062	-0.325	-0.701	1.064	-0.420	[49](syst)
²⁵⁹ Md	-0.142	0.627	-0.019	1.242	-	[49](syst)
$\langle\delta Q_\alpha\rangle_{\text{rms}} =$	0.643	0.647	0.593	1.148	0.450	

TABLE III: The $\log_{10}T_\alpha$ for superheavy nuclei found with the PS formula [34], Eq. (14). Meaning of the upper labels for experimental data is the same as in Table I. The stars indicate three cases, where the decay is mixed and the total lifetime is given. The percent number shows the weight of the α -transition.

Nuclei	FaNDF ⁰ (surf)	FaNDF ⁰ (vol)	SLy4	SkM*	MMM	$\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$	exp
²⁹⁴ 118	-1.52	-1.60	-1.73	-1.65	-3.41	-2.75 (0.13)	-3.23 - (-2.71) ^{a)}
²⁹⁰ Lv	-1.73	-1 40	0.20	-1.63	-1.64	-1.43 (0.23)	-2.27 - (-1.99) ^{a)}
²⁸⁶ Fl	-1.68	-1 28	1.32	-3.73	-1.72	-0.47 (0.16)	-0.96 - (-0.77) ^{a*)} , [50%]
²⁹⁴ 117	1.67	1.94	-1.98	-0.52	-1.50	-0.94 (0.10)	-1.51 - (-0.84) ^{b)}
²⁹⁰ 115	-1.34	-0.79	1.25	-0.52	-0.15	0.39 (0.11)	-0.10 - 0.56 ^{b)}
²⁸⁶ 113	2.51	0.26	4.82	-1.37	1.05	1.61 (0.15)	0.62 - 0.91 ^{b)}
							0.60 - 1.11 ^{c)}
²⁸² Rg	-1.09	-1.54	0.23	-3.49	0.76	2.80 (0.10)	2.06 - 2.72 ^{b)}
²⁷⁸ Mt	-1.24	0.57	0.08	-1.52	0.96	0.84 (0.09)	0.34 - 1.00 ^{b)}
²⁷⁴ Bh	2.08	1.36	2.23	1.06	2.50	2.04 (0.10)	1.26 - 1.92 ^{b)}
²⁷⁰ Db	4.79	6.52	6.05	3.29	-	4.58 (0.12)	3.33 - 4.02 ^{b)}
²⁹³ 117	0.15	0.42	-0.81	-0.84	-2.15	-1.31 (0.12)	-1.41 - (-1.02) ^{d)}
²⁸⁹ 115	-2.33	-1.88	1.07	-0.69	-0.83	-0.26 (0.13)	-0.31 - 0.18 ^{d)}
²⁸⁵ 113	-0.23	0.09	1.69	-4.14	-0.07	0.43 (0.14)	0.69 - 1.08 ^{c)}
²⁹¹ Lv	-0.924	-0.995	0.12	-0.52	-0.801	-0.75 (0.18)	-1.92 - (-1.40) ^{a)}
²⁸⁷ Fl	-0.269	0.325	3.65	-2.89	-0.524	0.55 (0.17)	-0.43 - (-0.19) ^{a)}
²⁸³ Cn	-1.02	-0.409	-1.00	-3.93	-0.098	1.29 (0.18)	0.49 - 0.70 ^{a)}
²⁷⁹ Ds	-3.09	-2.57	-0.95	-3.70	-0.962	0.13 (0.18)	-0.80 - (-0.60) ^{a*)} , [10%]
²⁷⁵ Hs	0.955	0.120	0.14	-0.76	0.690	0.60 (0.18)	-0.92 - (-0.39) ^{a)}
²⁷¹ Sg	2.03	4.20	2.67	2.71	2.15	2.29 (0.17)	1.89 - 2.41 ^{a*)} , [70%]
²⁸⁸ 115	-1.97	-0.97	3.53 -2.23	-0.93	-0.04 (0.16)	-1.24 - (-0.72) ^{e)}	
²⁸⁴ 113	-0.81	-0.62	-0.10 -3.83	-0.86	0.57 (0.17)	-0.51 - 0.03 ^{e)}	
²⁸⁰ Rg	-2.79	-2.94	-2.34 -3.95	-1.72	0.70 (0.17)	0.36 - 0.90 ^{e)}	
²⁷⁶ Mt	-2.13	-1.39	-1.64 -2.71	-0.58	0.08 (0.17)	-0.33 - 0.20 ^{e)}	
²⁷² Bh	2.02	0.09	2.48 1.54	1.69	1.88 (0.19)	0.80 - 1.33 ^{e)}	
$\langle \delta \lg T \rangle_{\text{rms}} =$	1.52	1.53	1.89	2.46	0.70	0.33	

to penetration through the centrifugal barrier. On the other hand, a favored transition may occur to excited state of the daughter nucleus.

An optimum set of the parameters of Eq. (13) for uranium and trans-uranium regions can be found in Ref. [34], where authors modified this formula to five-parameter PS form:

$$\log_{10}T_\alpha(Z, N) = aZ [Q_\alpha(Z, N) - \bar{E}_i]^{-1/2} + bZ + c, \quad (14)$$

with the following set of parameters $a = 1.5372$, $b = -0.1607$ and $c = -36.573$. The parameter \bar{E}_i in (14) has the meaning of the average excitation energy of the daughter nucleus, being zero in the case of even-even nuclei. For other types of nuclei, the following values of

parameters were found in [34]:

$$\begin{aligned} \bar{E}_i &= \bar{E}_p = 0.113 \text{ MeV} \quad \text{for odd-proton,} \\ \bar{E}_i &= \bar{E}_n = 0.171 \text{ MeV} \quad \text{for odd-neutron,} \\ \bar{E}_i &= \bar{E}_p + \bar{E}_n \quad \text{for odd-odd} \end{aligned} \quad (15)$$

nuclei. These rather simple and transparent PS formulas were used in [14] and they are also used in the present work. The results are given in Table III. The meaning of superscripts for experimental values is the same as in Table I. For three parent nuclei, namely ²⁸⁶Fl, ²⁷⁹Ds, and ²⁷¹Sg, there is a strong competition between α -decay and fission. In these cases, we gave the total lifetime values only and the α -decay percentage only, as it was given in the original experimental works. Recently, there has been several theoretical analysis of such competition, see e.g. [55] and references therein.

Since the formula (15) for T_α is essentially empirical, it is reasonable to examine some alternative for it. The empirical formula for T_α of favored α -transitions by Royer [54], or its modification by Royer and Zhang [35], has the form of

$$\log_{10}T_\alpha(Z, N) = a + bA^{1/6} + \frac{cZ}{\sqrt{Q_\alpha}}. \quad (16)$$

TABLE IV: The coefficients [35] of the RZ formula (16).

	a	b	c
ee	-25.31	-1.1629	1.5864
eo	-26.65	-1.0859	1.5848
oe	-25.68	-1.1423	1.592
oo	-29.48	-1.113	1.6971

TABLE V: The $\log_{10}T_\alpha$ values for superheavy nuclei found with the RZ formula [35], Eq. (16). Meaning of the upper labels for the experimental data is the same as in Table I.

Nuclei	FaNDF ⁰ (surf)	FaNDF ⁰ (vol)	SLy4	SkM*	MMM	$\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$	exp
²⁹⁴ 118	-2.14	-2.22	-2.35	-2.27	-4.09	-3.41 (0.14)	-3.23 - (-2.71) ^{a)}
²⁹⁰ Lv	-2.34	-2.00	-0.34	-2.23	-2.25	-2.02 (0.20)	-2.27 - (-1.99) ^{a)}
²⁸⁶ Fl	-2.27	-1.85	0.83	-4.39	-2.30	-1.02 (0.16)	-0.96 - (-0.77) ^{a*)} , [50%]
²⁹⁴ 117	1.57	1.86	-2.30	-0.75	-1.79	-1.19 (0.11)	-1.51 - (-0.84) ^{b)}
²⁹⁰ 115	-1.65	-1.07	1.09	-0.78	-0.38	0.19 (0.11)	-0.10 - 0.56 ^{b)}
²⁸⁶ 113	2.39	2.44	4.83	-1.71	0.86	1.44 (0.16)	0.62 - 0.91 ^{b)}
							0.60 - 1.11 ^{c)}
²⁸² Rg	-1.44	-1.92	-0.05	-3.99	0.51	2.67 (0.11)	2.06 - 2.72 ^{b)}
²⁷⁸ Mt	-1.62	0.29	-0.23	-1.92	0.69	0.57 (0.09)	0.34 - 1.00 ^{b)}
²⁷⁴ Bh	1.85	1.09	2.01	0.77	2.29	1.81 (0.10)	1.26 - 1.92 ^{b)}
²⁷⁰ Db	4.66	6.47	5.98	3.10	3.23	4.45 (0.12)	3.33 - 4.02 ^{b)}
²⁹³ 117	-0.33	-0.05	-1.30	-1.34	-2.67	-1.82 (0.13)	-1.41 - (-1.02) ^{d)}
²⁸⁹ 115	-2.85	-2.39	0.62	-1.18	-1.31	-0.73 (0.13)	-0.31 - 0.18 ^{d)}
²⁸⁵ 113	-0.70	-0.38	1.26	-4.68	-0.53	-0.03 (0.14)	0.69 - 1.08 ^{c)}
²⁹¹ Lv	-1.22	-1.30	-0.18	-0.81	-1.10	-1.05 (0.18)	-1.92 - (-1.40) ^{a)}
²⁸⁷ Fl	-0.57	0.02	3.36	-3.22	-0.83	0.25 (0.17)	-0.43 - (-0.19) ^{a)}
²⁸³ Cn	-1.33	-0.72	-1.32	-4.27	-0.41	0.98 (0.18)	0.49 - 0.70 ^{a)}
²⁷⁹ Ds	-3.43	-2.90	-1.27	-4.04	-1.29	-0.19 (0.17)	-0.80 - (-0.60) ^{a*)} , [10%]
²⁷⁵ Hs	0.63	-0.20	-0.19	-1.08	0.37	0.28 (0.18)	-0.92 - (-0.39) ^{a)}
²⁷¹ Sg	1.71	3.88	2.35	2.39	1.83	1.96 (0.27)	1.89 - 2.41 ^{a*)} , [70%]
²⁸⁸ 115	-2.29	-1.22	3.54	-2.56	-1.17	-0.60 (0.17)	-1.24 - (-0.72) ^{e)}
²⁸⁴ 113	-1.08	-0.88	-0.33	-4.29	-1.13	0.38 (0.18)	-0.51 - 0.03 ^{e)}
²⁸⁰ Rg	-3.20	-3.37	-2.73	-4.44	-2.07	0.49 (0.18)	0.36 - 0.90 ^{e)}
²⁷⁶ Mt	-2.53	-1.75	-2.02	-3.15	-0.89	-0.19 (0.18)	-0.33 - 0.20 ^{e)}
²⁷² Bh	1.82	-0.21	2.30	1.32	1.48	1.68 (0.20)	0.80 - 1.33 ^{e)}
$\langle \delta \lg T \rangle_{\text{rms}} =$	1.67	1.64	1.87	2.82	0.87	0.23	

Here the coefficients a , b , c are different for four different kinds of nuclei, see Table IV. The abbreviation ‘eo’ means even Z , odd N , and so on.

The unfavored α -transitions occur in odd and odd-odd nuclei, provided that the J^π values of the parent and daughter nuclei do not coincide, resulting α -particle orbital moment l being nonzero. Several generalizations of Eq. (16), which take into account additional hindrance due to the contribution of the centrifugal barrier, have been presented [56–59]. Here, we use a simple parameter free ansatz for this l -dependent addendum by Dong *et al.* [59]:

$$\log_{10}T_\alpha(l) = \log_{10}T_\alpha(l=0) + \frac{l(l+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}}. \quad (17)$$

For the first term of this formula the RZ recipe, Eq. (16), was used.

Recently Wang *et al.* [60] used a modification of Eq. (17), with four additional parameters for the l -dependent term. In this work, the role of the centrifugal term was examined in detail. In the sample of 341 α -transitions, with J^π values known from [56] for both the parent and

daughter nuclei, the average difference

$$\langle \delta \lg T \rangle_{\text{rms}} = \sqrt{\sum_i (\log_{10}T_\alpha^{\text{theor}} - \log_{10}T_\alpha^{\text{exp}})^2 / N} \quad (18)$$

between the theoretical predictions (with the experimental values of Q_α used) and the experimental values for the lifetimes were found in different models. At first, the initial Royer formula (16) was used and then, the one (17) with the l -dependent term included. It turned out that the gain for the due to the $\langle \delta \lg T \rangle_{\text{rms}}$ value is about 0.1. To be more exact, from $\langle \delta \lg T \rangle_{\text{rms}} = 0.587$ without the l -term to $\langle \delta \lg T \rangle_{\text{rms}} = 0.481$ with. Of course, this gain is different for different kinds of nuclei. It is equal to zero for even-even nuclei, being about 0.2 for odd-odd ones. The use of the modified centrifugal term in [60] results in a rather small additional gain, $\langle \delta \lg T \rangle_{\text{rms}} = 0.433$. The corresponding values of $\langle \delta \lg T \rangle_{\text{rms}} = 0.536$ in [57] and $\langle \delta \lg T \rangle_{\text{rms}} = 0.561$ in [58] are also given in [60]. The predictions for lifetimes found in this work with the use of different theoretical values of Q_α are essentially worse. For example, for the Skyrme HFB27 EDF the average deviation from the experiment $\langle \delta \lg T \rangle_{\text{rms}} = 1.617$ was found. The values of $\langle \delta \lg T \rangle_{\text{rms}}$ found with Eq. (18) for each theoretical column in Table III are given in the last

line of this table. In this calculation, we excluded three cases of mixed decays.

In contrast to [60], present work addresses odd and odd-odd superheavy nuclei for which characteristics are often not known from the experiment. Therefore, it is reasonable to use for systematic calculations of the $\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$ values Eq. (17) assuming that all the α -transitions are favored, i.e. putting $l = 0$. Thus, we use, in fact, the RZ formula (16). In Table V, we present the results of such calculations for the same set of nuclei as in Table III. In both of the tables, we found from Eq. (18) for each kind of the theory used the average difference between the theoretical prediction for $\log_{10}T_\alpha$ and the corresponding experimental value, three cases of mixed decays again being excluded from the averaging procedure. Comparison of the values of $\langle \delta \lg T \rangle_{\text{rms}}$ in the columns of $\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$ in tables III and V is a direct comparison of accuracy of Eq. (15) from [34] and Eq. (16) from [35]. We see that the latter is a bit more accurate that is not strange as it contains twelve fitted parameters in comparison with five ones in the first case.

We can now compare accuracy of different theoretical methods in description of the α -decay lifetimes. We see that both the methods to calculate the lifetimes leads to approximately the same accuracy, the Parkhomenko–Sobiczewski method turning out to be a bit better. Evidently, the accuracy of the RZ method could be made higher if all the α -transitions were not considered as a favored. Again, the MMM approach is more accurate than all the self-consistent methods used. For this characteristic, the Fayans method with the FaNDF⁰ EDF and both models for pairing exceeds in accuracy a bit the SLy4 EDF and significantly the SkM* one. Note that the $\langle \delta \lg T \rangle_{\text{rms}}$ value for the Fayans EDF is approximately the same as that found in [60] for the HFB-27 EDF, the member of the family of most accurate in predicting nuclear masses Skyrme EDFs.

In order to estimate the role of the l -dependent term in Eq. (17) we chose three α -decays in the chain of $^{293}\text{117}$ for which we obtained $\log_{10}T_\alpha(Q_\alpha^{\text{exp}}) < \log_{10}T_\alpha^{\text{exp}}$. Due to hindrance effect of the centrifugal barrier, its inclusion may improve the agreement with the data. The calculated results are given in Table VI. We see that, indeed, the inclusion of the l -dependent term does improve the situation with the use of Eq. (17). The values of $\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$ agree with the data at $l = 3, l = 4$ for the $^{293}\text{117}$ and $^{289}\text{115}$ parent nuclei and at $l = 4, l = 5$ for the $^{285}\text{113}$ one. Unfortunately, this remedy can help only in the cases where the inequality $\log_{10}T_\alpha(Q_\alpha^{\text{exp}}) < \log_{10}T_\alpha^{\text{exp}}$ holds, whereas the opposite sign of this inequality takes place in many cases in Table V. For example, this happens for the $^{275}\text{108}$ nucleus for which we obtained $\log_{10}T_\alpha(Q_\alpha^{\text{exp}}) - \log_{10}T_\alpha^{\text{exp}} = 0.487$. However, as the analysis in [60] showed the average accuracy of Eq. (17) and analogous formulas in [60] for $\log_{10}T_\alpha$ is of the order of 0.5, therefore the disagreement under discussion is not extraordinary.

TABLE VI: The role of the l -dependent term in formula (17). Meaning of the upper labels for the experimental data is the same as in Table I.

Nuclei	1	$\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$	exp
$^{293}\text{117}$	0	-1.82	-1.41 - (-1.02) ^{d)}
	1	-1.74	
	2	-1.60	
	3	-1.38	
	4	-1.09	
	5	-0.72	
$^{289}\text{115}$	0	-0.73	-0.31 - 0.18 ^{d)}
	1	-0.66	
	2	-0.51	
	3	-0.29	
	4	0.01	
	5	0.37	
$^{285}\text{113}$	0	-0.03	0.69 - 1.08 ^{c)}
	1	0.05	
	2	0.20	
	3	0.42	
	4	0.72	
	5	1.09	

IV. CONCLUSION

Alpha-decay energies Q_α for several chains of super-heavy nuclei are found by employing the Fayans functional FaNDF⁰. Two models for the effective pairing force were used, the volume and the surface pairing. The results are compared to the experimental data and predictions of two Skyrme functionals, SLy4 [50] and SkM* [51]. Predictions of the macro-micro method [10, 11] are also considered. The Fayans EDF results in the average deviation from experimental energies by $\langle \delta Q_\alpha \rangle_{\text{rms}}^{\text{tot}} = 0.643$ MeV with the surface pairing and 0.647 MeV with the volume pairing. These values are slightly larger than the corresponding SLy4 value of 0.593 MeV but significantly less than the SkM* value of 1.148 MeV. However, in this problem all the self-consistent methods considered give in the MMM, the corresponding value being only 0.454 MeV. It is worth to stress that the FaNDF⁰ EDF used here was found in [23] by adjusting to masses and radii of spherical nuclei only, from the calcium to the lead region. Therefore a readjustment of its parameters with the use of all the nuclear chart is desirable.

The corresponding α -decay lifetimes are calculated with the use of the semi-phenomenological five-parameter PS formula [34], or, alternatively, the twelve-parameter RZ one [35]. The role of the l -dependent term for the unfavored α -transitions in the form of [59] was also examined. Accuracy of the method itself to calculate the lifetimes can be estimated by comparing the $\log_{10}T_\alpha(Q_\alpha^{\text{exp}})$ values with the experimental data of $\log_{10}T_\alpha$. For the bulk of 24 α -transitions between super-heavy nuclei we examined, the RZ formula gives an average deviation of $\langle \delta \lg T \rangle_{\text{rms}} = 0.23$, whereas it is equal to 0.33 for the PS

case. Thus, the first method looks more accurate itself. However, the accuracy of all calculations with the use of theoretical Q_α values turns out a bit worse in the RZ case.

Any inaccuracy in finding Q_α leads to a defect in reproducing the T_α values, the scale of disagreement being even enhanced. Again, the the MMM exceeds in accuracy all the self-consistent calculations. The corresponding value of $\langle \delta \lg T \rangle_{\text{rms}}$ is equal to 0.69 with the use of the PS formula and 0.87, with the RZ one. In this problem the Fayans approach have some advantage over the other self-consistent methods used. For this, we obtained $\langle \delta \lg T \rangle_{\text{rms}} \simeq 1.50$ with the PS formula and $\simeq 1.65$ for the RZ one. For comparison, the corresponding values there are 1.89 and 1.87 for the SLy4 EDF, and also 2.46 and 2.82 for the SkM* one. Note that the average deviation from experiment for the Skyrme HFB-27 EDF found in [60] is $\langle \delta \lg T \rangle_{\text{rms}} \simeq 1.5$, i.e. it is comparable to that for the Fayans EDF.

In conclusion, we should again stress that readjustment of the Fayans EDF parameters with the use of wider bulk of nuclei, including the deformed and heavy ones, is nec-

essary to attempt to reach the level of accuracy in reproducing the Q_α and T_α values comparable to that of the MMM.

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